

## 2006-2007 BHS Math Club Warm-Up 5: Algebra

Name \_\_\_\_\_ Grade \_\_\_\_\_

1. Factor completely:  $x^4 - y^2$ .
2. The sum of the roots of a quadratic is 13. The product of the roots is 37. What is one possible quadratic?
3. Evaluate  $10001 \cdot 1337 \cdot 100000001$ .
4. Reduce  $\frac{27x^3 + 8}{3x + 2}$ .
5. Determine the maximum value of the function  $g(x) = -x^2 - 8x - 7$ .
6. If  $f(x) = \frac{x + 1}{x - 1}$ , evaluate  $f(f(f(f(f(1337))))))$ .
7. Solve the system of equations for  $(a, b, c, d)$ :  $a + 3b = 10$ ,  $b + 3c = 12$ ,  $3c + 7d = 58$ ,  $3d + 7a = 28$ .
8. Find the largest coefficient in the expansion of  $(2x + 3y)^5$ .
9. Let  $f$  be a function satisfying  $f(xy) = f(x) + f(y)$  for all positive  $x, y$ . If  $f(2) = 1$ , what is  $f(2^{1337})$ ?
10. Find the largest prime factor of  $17^3 - 64$ .
11. Let  $\alpha, \beta, \gamma, \delta$  be the roots of the quartic  $x^4 - 4x^2 + 3x + 8 = 0$ . Find  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 - \alpha\beta\gamma\delta$ .
12. Evaluate  $\sum_{a=1}^1 \sum_{b=1}^3 \sum_{c=1}^3 \sum_{d=1}^7 bdac$ .
13. Determine the units digit in the decimal expansion of  $(20 + \sqrt{397})^{2674}$ .
14.  $a, b, c, x, y, z$  are real numbers such that  $a^3 + xa^2 + ya + z = 0$ ,  $b^3 + xb^2 + yb + z = 0$ , and  $c^3 + xc^2 + yc + z = 0$ . If  $a, b, c$  are distinct, find  $a^3 + b^3 + c^3 - 3abc$  in terms of  $x, y, z$ .
15. Let  $P(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$  be a monic polynomial of degree  $n$  where  $n$  is odd and  $n \geq 3$ . Let  $C$  be a positive real and  $i, j$  be integers with  $0 \leq i, j \leq n - 1$  such that  $Ca_i^2 < a_j$  is a sufficient condition for  $P(x)$  to have at least one nonreal root. Evaluate  $C \cdot i \cdot j$  in terms of  $n$ .