

2006-2007 BHS Math Club Warm-Up 3

Name _____ Grade _____

1. $\triangle ABC$ has area 6 and perimeter 12. If all of its sides have integer length, what is the length of the longest side?
2. I have a bag with 2 red marbles, 3 blue marbles, and 50 green marbles. How many marbles must I take out to guarantee that I have at least one of each color?
3. Simplify: $\frac{a^2 + ab + a^2c + abc}{b^2 + ab + b^2c + abc}$.
4. Let $f(x) = x^2$ and $g(x) = \frac{1}{x^3}$. Evaluate $g^{-1}(f(8))$.
5. Andreea wants her name in a 15-question test. If it costs \$1 to have her name in the 1st question, \$2 for the 2nd question, ..., and \$15 for the 15th question, how much money would it cost to have her name in every prime numbered question?
6. What is the maximum number of squares you can select in a 5×5 grid such that no two squares lie in the same row, column, or diagonal?
7. How many times do the graphs of $y = e^x$ and $y = |x|$ intersect?
8. Evaluate $\frac{2}{2^2 - 1} + \frac{2}{3^2 - 1} + \cdots + \frac{2}{6^2 - 1}$.
9. What is the sum of the coefficients of $(x + 2y - z)^3$?
10. Which is greater: (a) the volume of a cube of side length $\frac{22}{7}$ or (b) the area of a circle of radius $\frac{22}{7}$?
11. Let n be a three-digit number such that when you reverse the digits of n you get a number that is $\frac{53}{86}$ times as big as n . What is n ?
12. Find the number of solutions to $x + y + z = 12$ for positive integers x, y, z .
13. What is the area of the shape formed by connecting the solutions to $x^3 - 5x^2 + 11x - 7 = 0$ in the complex plane?
14. You have 10 problems you want to put on a test; 5 are easy and 5 are hard. You want to write the test such that, if you go in order, at any point on the test you will never have done more hard problems than easy ones. How many ways are there to do this?
15. How many elements are in the smallest subset A of $S = \{1, 2, \dots, 100\}$ such that for every $n \in S$ there exist $a, b \in A \cup \{0\}$ (not necessarily distinct) such that $a + b = n$?