2007 Mock AMC 12

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Scoring: Correct +6 Blank +1.5 Wrong +0
Time: 75 minutes
Calculators Permitted

1. You bought ten total boxes of milk chocolate and white chocolate. If you bought four more boxes of milk chocalate than boxes of white chocolate, how many boxes of milk chocolate did you buy?

(A) 0 (B) 3 (C) 5 (D) 7 (E) 10

2. Define $x \odot y = 2x - y^2$. What is $(a^2 \odot a) \odot a$?

(A) $-a^2$ (B) $-a$ (C) 0 (D) a (E) a^2

3. You have three cards, each with a number on it. If you remove any one card, the possible sums of the numbers on the other two cards are 5, 10, and 17. What is the sum of all three numbers?

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20

4. Order from least to greatest: $X = 10\%$ of 100, $Y = 11\%$ of 90, $Z = 12\%$ of 80.

(A) X, Y, Z (B) X, Z, Y (C) Y, Z, X (D) Y, X, Z (E) Z, Y, X

5. How many whole 3×3 squares can you fit in a 10×10 square?

(A) 3 (B) 6 (C) 9 (D) 11 (E) 27

6. Your friend is 15 miles east from you. If you are traveling north at 12 miles per hour and he is traveling at 13 miles per hour, what is the minimum time it takes for him to meet you?

(A) 2 hours (B) $\frac{5}{2}$ hours (C) 3 hours (D) $\frac{7}{2}$ hours (E) 4 hours

7. The 1×1 bases of six right square pyramids with height 1 are attached to the faces of a cube of side length 1. The resulting solid has how many edges (E), faces (F), and vertices (V)?

(A) 32 E, 20 F, 14 V (B) 32 E, 20 F, 18 V (C) 32 E, 24 F, 14 V (D) 36 E, 24 F, 14 V (E) 36 E, 32 F, 6 V

8. Three boys, Al, Bob, and Chuck, and three girls, Amy, Beatrice, and Christina, want to sit in a row of six chairs. Each boy wants to sit to the left of a girl, but not the girl whose name begins with the same letter as his own. In how many ways can they sit down like this?

(A) 1 (B) 2 (C) 3 (D) 6 (E) 12

9. Jack is 20 years older than Jill. If Jill's age divides Jack's age, what is the sum of all the possible values of Jill's age?

(A) 42 (B) 41 (C) 31 (D) 22 (E) 21

10. A *nearly-perfect* number is a positive integer $n > 1$ such that the sum of its positive divisors, excluding itself, is $n - 1$. What is the sum of the first three *nearly-perfect* numbers?

(A) 12 (B) 14 (C) 26 (D) 28 (E) 30

11. How many integers satisfy the equation $|x+7|+|x-7|=14$?

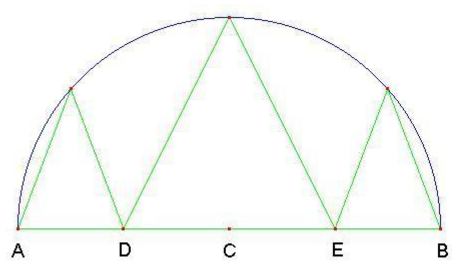
(A) 1 (B) 7 (C) 8 (D) 14 (E) 15

12. You have a digital counter that counts the number of people who enter and exit a room. Initially, it reads 0000 and the room is empty. When the first person enters the room, the counter increments by 1 to 0001. Strangely, when the second person enters, the counter increments by 2 to 0003. After that, each time a person enters, you find that the counter increments by the twice as much as it did the previous time. When people exit, however, the counter always decrements by 1. What is the smallest number greater than 2007 that the counter can read?

(A) 2025 (B) 2035 (C) 2036 (D) 2037 (E) 2047

13. Let AB be the diameter of a semicircle and C be the midpoint of AB. Let D and E be the midpoints of AC and CB, respectively. Form an isosceles triangle with AD as the base and third vertex on the semicircle. Do the same with DE and EB, forming three isosceles triangles total. If $AB=8$, what is sum of the areas of these three triangles? See diagram below.

(A) 12 (B) $4+2\sqrt{7}$ (C) $4+4\sqrt{3}$ (D) $8+2\sqrt{7}$ (E) $8+4\sqrt{3}$



14. An odd number of students took a math test worth 100 points. The median score was 80. If each student got a different integer score and the mean of the scores was 87 points, what is the minimum number of students who took the test?

(A) 5 (B) 7 (C) 9 (D) 11 (E) 13

15. A bug makes an infinite sequence of moves along a number line, starting from zero. It moves one unit in the positive direction. From then on, on the nth move, the bug moves 2^{1-n} units in the direction opposite its previous move. Where does the bug end up?

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) 1

16. If $a_0=2007$ and $a_n=\sqrt[2007]{(a_{n-1})^{2007}-2007}$ for $n\geq 1$, what is the smallest n for which a_n is not positive?

(A) 2006 (B) 2007 (C) 2007^{2006} (D) 2006^{2007} (E) 2007^{2007}

17. Find the area of the region containing points (x,y) which satisfy $1+\log\log|x|\leq 0$ and $1+\log\log|y|\leq 0$.

(A) $4\cdot 10^{\frac{1}{5}}$ (B) $4\cdot(10^{\frac{1}{10}}-1)^2$ (C) $4\cdot 10^{20}$ (D) $4\cdot(10^{10}-1)^2$ (E) 10^{10}

18. If you randomly paint the faces of a cube with three colors, what is the probability that no two faces sharing an edge are the same color?

(A) $\frac{1}{3^4}$ (B) $\frac{2}{3^5}$ (C) $\frac{1}{3^5}$ (D) $\frac{2}{3^6}$ (E) $\frac{1}{3^6}$

19. If $(1+\sqrt{3})\sin x + (1-\sqrt{3})\cos x = 0$, then which of the following must be true?

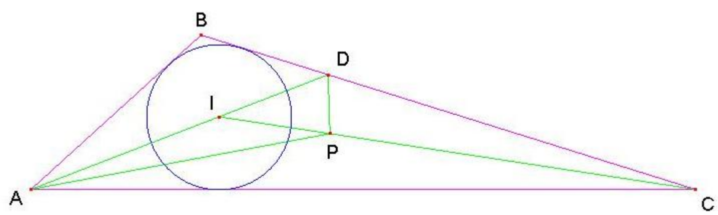
(A) $\sin x = -\cos x$ (B) $\sin x \cos x = \frac{1}{4}$ (C) $\tan x = \sqrt{3}$ (D) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1}{\sqrt{3}}$ (E) None of them.

20. Let A and B be points on the parabola $y = x^2$. If the line passing through A and B has x-intercept -12 and the sum of the x-coordinates of A and B is 2, what is the length of the segment AB?

(A) $8\sqrt{5}$ (B) $10\sqrt{5}$ (C) $12\sqrt{5}$ (D) $14\sqrt{5}$ (E) $16\sqrt{5}$

21. A circle with center I is inscribed in $\triangle ABC$. Extend AI to intersect side BC at point D, and let P be the intersection of IC with the angle bisector of $\angle IAC$. If $\angle A = 40°$ and $\angle B = 110°$, find the measure of $\angle APD$ in degrees. See diagram below.

(A) 95° (B) 100° (C) 105° (D) 110° (E) 115°



22. Suppose you have ten coins, each of which has a heads side and a tails side, arranged in a line. A move consists of flipping any two coins which are adjacent to each other. If you start out with an even number of coins facing heads up, at most how many moves does it take to make all the coins face heads up?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

23. Let C be a circle of radius 1. Let R be the union of the interiors of all isosceles right triangles with legs of length $\sqrt{2}$ and exactly two vertices on C. What is the area of R?

(A) $5\pi - 2$ (B) $\frac{9\pi}{2}$ (C) $(2+2\sqrt{2})\pi$ (D) 5π (E) $\left(\frac{5}{2}+2\sqrt{2}\right)\pi$

24. The lengths of the sides of a triangle are the roots of the equation $7x^3 + 35x = 28x^2 + 13$. If the square of the area of the triangle is $\frac{p}{q}$, where p and q are relatively prime positive integers, what is $p+q$?

(A) 9 (B) 11 (C) 12 (D) 13 (E) 15

25. Let S be the set of reals of the form $\sin\left(\frac{2007\pi}{n}\right)$, where n is any positive integer greater than 2007. How many ordered pairs (a, b) exist such that a and b are both elements of S, not necessarily distinct, and $a^2 + b^2 = 1$?

(A) 45 (B) 48 (C) 68 (D) 72 (E) 117