# Mock AIME 6

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1. Find the remainder when $11^{2005}$ is divided by 1000.

2. Suppose $\log_x y = 3$, $\log_y z = 11$, and $\log_z w = 29$. Given that $z = 1024y$ and $wx = p^q$, where $p$ is a prime, find $3q$.

3. Suppose there are 5 red points and 4 blue points on a circle. Let $\frac{m}{n}$ be the probability that a convex polygon whose vertices are among the 9 points has at least one blue vertex, where $m$ and $n$ are relatively prime. Find $m + n$.

4. Let $O$ be the circumcircle of $\triangle ABC$ and let $D$, $E$, and $F$ be points on sides $BC$, $AC$, and $AB$, respectively, such that $CD = 8$, $BD = 6$, $BF = 3$, $AF = 2$, and $AE = 4$, and $AD$, $BE$, and $CF$ are concurrent. Let $H$ be a point on minor arc $BC$. Extend $AC$ beyond to $C$ to $G$ such that $GH$ is tangent to the circle at $H$ and $GC = 12$. Find $GH^2$.

5. Let $f(x) = \sqrt{x + 2005\sqrt{x + 2005\sqrt{\ldots}}}$ where $f(x) > 0$. Find the remainder when

$$f(0)f(2006)f(4014)f(6024)f(8036)$$

is divided by 1000.

6. Let $z_0 = \cos\frac{\pi}{2005} + i\sin\frac{\pi}{2005}$ and $z_{n+1} = (z_n)^{n+1}$ for $n = 0, 1, 2, \ldots$. Find the smallest $n$ such that $z_n$ is an integer.

7. Let $R$ be the region inside the graph $x^2 + y^2 - 14x + 4y - 523 = 0$ and to the right of the line $x = -5$. If the area of $R$ is in the form $a\pi + b\sqrt{c}$ where $c$ is squarefree, find $a + b + c$.

8. If $x\sqrt[3]{4} + \frac{\sqrt[3]{2}}{x} = 3$ and $x$ is real, evaluate $64x^9 + \frac{8}{x^9}$.

9. You have 5 boxes and 2005 balls. 286, 645, and 1074 of these balls are blue, green, and red, respectively. Suppose the boxes are numbered 1 through 5. You place 1 blue ball, 3 green balls, and 3 red balls in box 1. Then 2 blue balls, 5 green balls, and 7 red balls in box 2. Similarly, you put $n$ blue balls, $2n+1$ green balls, and $4n-1$ red balls in box $n$ for $n = 3, 4, 5$. Repeat the entire process (from boxes 1 to 5) until you run out of one color of balls. How many red balls are in boxes 3, 4, and 5? (NOTE: After placing the last ball of a certain color in a box, you still place the balls of the other colors in that box. You do not, however, place balls in the following box.)

10. There are two ants on opposite corners of a cube. On each move, they can travel along an edge to an adjacent vertex. If the probability that they both return to their starting position after 4 moves is $\frac{m}{n}$, where $m$ and $n$ are relatively prime, find $m+n$. (NOTE: They do not stop if they collide.)

11. Evaluate $\displaystyle\sum_{n=0}^{44} \frac{\sin(4n+1)}{\cos(2n)\cos(2n+1)} + \sum_{n=46}^{90} \frac{\sin(4n-1)}{\cos(2n-1)\cos(2n)}$.

12. A rigged coin has the property that when it is flipped 2005 times the probability of getting heads 589 times is equal to the probability of getting heads 590 times. If $\frac{m}{n}$ is the probability of getting a two heads in a row, where $m$ and $n$ are relatively prime, find $m+n$.

13. In $\triangle ABC$, $\angle A = 30°$, $AC = 28\sqrt{3}$, and $AB = 42$. Let $E$ be on side $AC$ and $D$ be on side $AB$ such that $DE$ is perpendicular to $AC$ and $DE = 14$. If $N = [\triangle ECB] - [\triangle EDB]$, find $\lfloor N \rfloor$. (NOTE: $[\triangle XYZ]$ denotes the area of $\triangle XYZ$.)

14. Let $f$ be a polynomial of degree 2005 with leading coefficient 2006 such that for $n = 1, 2, 3, \ldots, 2005$, $f(n) = n$. Find the number of 0's at the end of $f(2006) - 2006$.

15. An AIME has 15 questions, 5 of each of three difficulties: easy, medium, and hard. Let $e(X)$ denote the number of easy questions up to question $X$ (including question $X$). Similarly define $m(X)$ and $h(X)$. Let $N$ be the number of ways to arrange the questions in the AIME such that, for any $X$, $e(X) \geq m(X) \geq h(X)$ and if a easy and hard problem are consecutive, the easy always comes first. Find the remainder when $N$ is divided by 1000.